Low intermediate scales in SUSY SO(10) GUTs with left-right symmetry

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with S.K. Majee, M.K. Parida, and U.Sarkar, hep-ph/0701109, *Phys. Rev.* **D75,** 075003 (2007).

Plan

Question: Can low energy parity restoration be consistent with Grand Unification?

SO(10) symmetry breaking to SM

Difficulty of having a light M_R

Some possible solutions

- Threshold corrections
- Higher dimension operators
- Additional multiplets

Conclusions

Symmetry breaking

$$SO(10) \xrightarrow{210\ (M_U)} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\xrightarrow{16 \text{ or } 126\ (M_R)} SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$\xrightarrow{10\ (M_Z)} SU(3)_C \times U(1)_Q$$

Parity restored above M_R*. Also D-parity:* $g_L = g_R$

Model I with 16, Model II with 126

At M_R:

$\chi_{\mathbf{L}}(\mathbf{1}, \mathbf{2}, \mathbf{1}, -\mathbf{1}) + \chi_{\mathbf{R}}(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\mathbf{1}) +$ C.C $\Leftarrow$ Model I

$\mathbf{\Delta_L}(\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{2}) + \mathbf{\Delta_R}(\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{2}) +$ C.C $\Leftarrow$ Model II

Symmetry breaking (Contd.)

Under Pati-Salam $\{SU(4)_c \times SU(2)_L \times SU(2)_R\}$ and $(SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L})$

$$\begin{aligned}
\mathbf{210} &= \{15,1,1\} + \{1,1,1\} + \underbrace{\{15,3,1\} + \{15,1,3\}}_{potential\ difficulty} + \ldots \\
&= (1,1,1,0) + (8,1,1,0) + (3,1,1,\frac{4}{3}) + (\bar{3},1,1,-\frac{4}{3}) + (1,1,1,0) + \ldots
\end{aligned}$$

$$\begin{aligned}
\mathbf{126} &= \{10,3,1\} + \{\overline{10},1,3\} + (15,2,2) + \ldots \\
&= (1,3,1,-2) + (1,1,3,2) + (6,3,1,\frac{2}{3}) + (6,1,3,-\frac{2}{3}) + \ldots \\
\mathbf{16} &= \{\bar{4},2,1\} + \{4,1,2\} \\
&= (1,2,1,1) + (1,1,2,-1) + (\bar{3},2,1,-\frac{1}{3}) + (3,1,2,\frac{1}{3})
\end{aligned}$$

$$\begin{aligned}
\mathbf{10} &= \{1,2,2\} + \{6,1,1\} \\
&= (1,2,2,0) + (3,1,1,-\frac{2}{3}) + (\bar{3},1,1,\frac{2}{3})
\end{aligned}$$

RG Equations

Two-step evolution: $M_Z \to M_R \to M_U$

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_R)} + \frac{a_i}{2\pi} \ln \frac{M_R}{M_Z} + \Theta_i - \Delta_i,$$

$$\frac{1}{\alpha_i(M_R)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i'}{2\pi} \ln \frac{M_U}{M_R} + \Theta_i' - \Delta_i' - \Delta_i^{(gr)},$$

$\Theta \Rightarrow$ two-loop, $\Delta \Rightarrow$ threshold corr., $\Delta^{(gr)} \Rightarrow$ Higher dim. op.

Only 1-loop, No threshold, No higher dim.

$\Rightarrow M_R \simeq 1.3 \times 10^{16}$ GeV, $\quad M_U \simeq 2.9 \times 10^{16}$ GeV

RG Equations: 2-loop

$$\Theta_i = \frac{1}{4\pi}\sum_j B_{ij}\ln\frac{\alpha_j(M_R)}{\alpha_j(M_Z)},$$

$$\Theta'_i = \frac{1}{4\pi}\sum_j B'_{ij}\ln\frac{\alpha_j(M_U)}{\alpha_j(M_R)},$$

$$B_{ij} = \frac{b_{ij}}{a_j},\ B'_{ij} = \frac{b'_{ij}}{a'_j}.$$

β function coefficients: $a_i \Rightarrow$ 1-loop, $b_{ij} \Rightarrow$ 2-loop

RG Equations: Threshold, Higher dim.

$$\Delta_i^{(S)} = \frac{1}{2\pi}\Sigma_\alpha b_i^\alpha \ln\frac{M^\alpha}{M_S} \equiv \frac{b_i}{2\pi}\ln\frac{M_i}{M_S}, \quad b_i = \Sigma_\alpha b_i^\alpha,$$

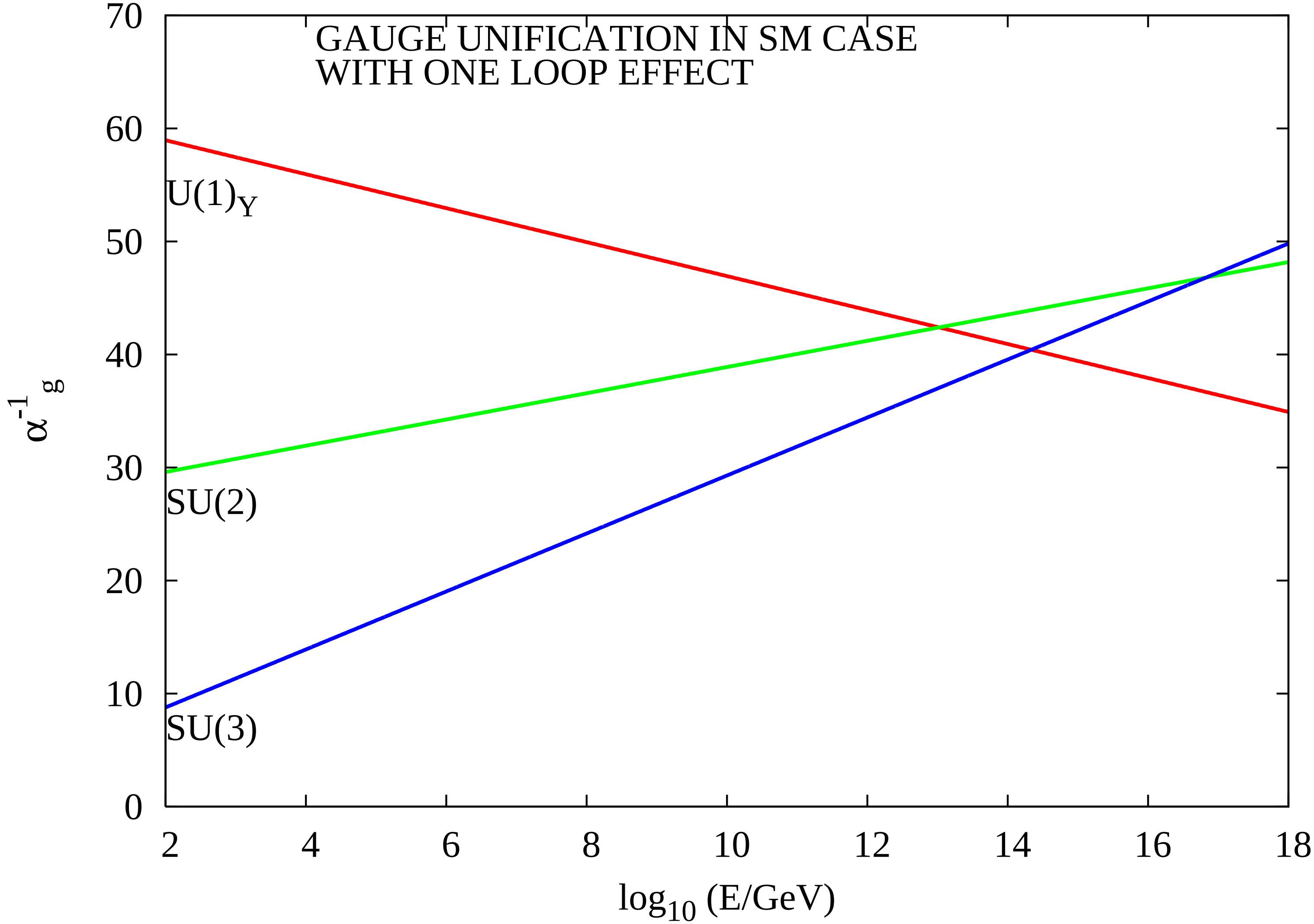
$$\Delta_i^{(R)} = \frac{1}{2\pi}\Sigma_\beta c_i^\beta \ln\frac{M^\beta}{M_R} \equiv \frac{b'_i}{2\pi}\ln\frac{M_i}{M_R}, \quad b'_i = \Sigma_\beta c_i^\beta,$$

$$\Delta'_i = \frac{1}{2\pi}\Sigma_\gamma d_i^\gamma \ln\frac{M^\gamma}{M_U} \equiv \frac{b''_i}{2\pi}\ln\frac{M_i}{M_U}, \quad b''_i = \Sigma_\gamma d_i^\gamma.$$

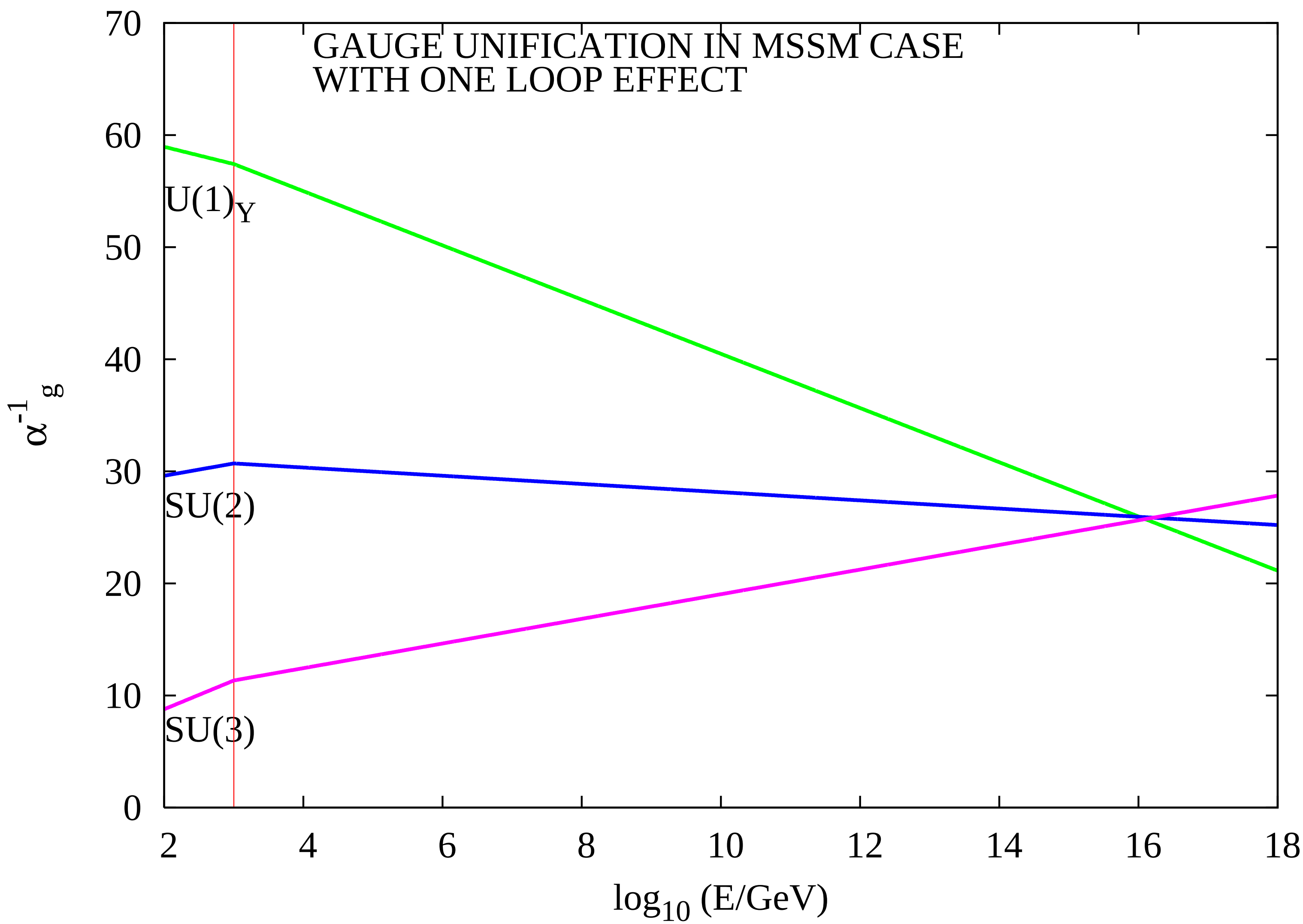
$$\Delta_i^{(gr)} = -\frac{\epsilon_i}{\alpha_G}, i = BL, 2L, 2R, 3C.$$

ϵ_i from non-renormalisable operators.

Gauge evolution in SM



Gauge evolution in MSSM



Beta function coeffs in LRS Model

Models I and II identical between M_Z and M_R:

$$\begin{pmatrix} a_Y \\ a_{2L} \\ a_{3C} \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix},$$

$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}, i, j \subset \mathcal{G}_{std}.$$

MSSM beta functions, set $M_{SUSY} = M_Z$

Beta function coefficients: 1-loop

Above M_R till M_U at one loop:

$$\begin{pmatrix} a'_{BL} \\ a'_{2L} \\ a'_{2R} \\ a'_{3C} \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \\ -3 \end{pmatrix}, \quad \text{Model I} \equiv \mathbf{16} + \overline{\mathbf{16}}$$

$$\begin{pmatrix} a'_{BL} \\ a'_{2L} \\ a'_{2R} \\ a'_{3C} \end{pmatrix} = \begin{pmatrix} 24 \\ 5 \\ 5 \\ -3 \end{pmatrix}, \quad \text{Model II} \equiv \mathbf{126} + \overline{\mathbf{126}}$$

Beta function coefficients: 2-loop

M_R to M_U two-loop coefficients:

$$b'_{ij} = \begin{pmatrix} \frac{23}{2} & \frac{27}{2} & \frac{27}{2} & 8 \\ \frac{9}{2} & 32 & 3 & 24 \\ \frac{9}{2} & 3 & 32 & 24 \\ 1 & 9 & 9 & 14 \end{pmatrix}, i,j \subset \mathcal{G}_{LR}. \quad \text{Model I} \quad \equiv \quad \mathbf{16} + \overline{\mathbf{16}}$$

$$b'_{ij} = \begin{pmatrix} 115 & 81 & 81 & 8 \\ 27 & 73 & 3 & 24 \\ 27 & 3 & 73 & 24 \\ 1 & 9 & 9 & 14 \end{pmatrix}, i,j \subset \mathcal{G}_{LR}. \quad \text{Model II} \quad \equiv \quad \mathbf{126} + \overline{\mathbf{126}}$$

RG results at 1-loop

Input:

$$\alpha_S(M_Z) = 0.1187,\ \alpha(M_Z) = 1/127.9,\ \sin^2\Theta_W = 0.2312$$

Model I $\equiv$ $\mathbf{16} + \overline{\mathbf{16}}$

$M_R^0 = 1.3 \times 10^{16}$ GeV, $\quad M_U^0 = 2.9 \times 10^{16}$ GeV. $\quad \alpha_G \simeq 1/24.25$

Model II $\equiv$ $\mathbf{126} + \overline{\mathbf{126}}$

$M_R^0 = 7.9 \times 10^{15}$ GeV, $\quad M_U^0 = 1.9 \times 10^{16}$ GeV. $\quad \alpha_G \simeq 1/24.00$

Parity restoration only at a high scale

Inclusion of two-loop effects pushes $M_R \to M_U$

Possibilties for amelioration

Threshold Corrections

Origin: Small splitting of masses around threshold, e.g., SUSY spectrum. For big-sized representations like $\mathbf{210}$ or $\mathbf{126} + \overline{\mathbf{126}}$ the GUT-scale threshold corrections are sizable.

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Non-renormalizable interactions

Gauge invariant but non-renormalizable interaction terms suppressed by inverse powers of the Planck scale may arise from quantum gravity.

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Additional light fi elds

Additional light multiplets can lower M_R.

GUT-scale threshold corrections

Superheavy components of $\mathbf{210} \oplus \mathbf{16} \oplus \overline{\mathbf{16}} \oplus \mathbf{10}$ (Model I) not exactly degenerate at M_U:

$$\Delta \ln \frac{M_R}{M_Z} = \frac{1}{7}\left[\frac{125}{3} \ln \frac{M_1}{M_U} - 106 \ln \frac{M_2}{M_U} + \frac{196}{3} \ln \frac{M_3}{M_U}\right],$$

$$\Delta \ln \frac{M_U}{M_Z} = \frac{1}{7}\left[\frac{25}{3} \ln \frac{M_1}{M_U} + 53 \ln \frac{M_2}{M_U} - \frac{196}{3} \ln \frac{M_3}{M_U}\right].$$

Enough room for significant lowering of M_R while keeping M_U within the Planck scale and with $10 \geq M_i/M_U \geq 0.1$.

Typical solutions

M_R (GeV)	M_U (GeV)	$\frac{M_1}{M_U}$	$\frac{M_2}{M_U}$	$\frac{M_3}{M_U}$	α_G^{-1}
10^{11}	1.2×10^{19}	$(1.48)^{-1}$	1.48	$(1.48)^{-1}$	23.7
10^{9}	10^{18}	0.272	1.770	0.831	23.7
10^{7}	10^{18}	0.158	1.950	0.832	23.7
10^{7}	5×10^{16}	0.151	2.750	1.524	27.7
10^{5}	5×10^{18}	0.180	3.30	1.076	26.7
10^{3}	10^{19}	0.154	4.760	1.301	28.7

GUT-scale threshold effect solutions (doublet model).

GUT-scale threshold corrections

Superheavy components of $\mathbf{210} \oplus \mathbf{126} \oplus \overline{\mathbf{126}} \oplus \mathbf{10}$ (Model II) not exactly degenerate at M_U:

$$\Delta \ln \frac{M_R}{M_Z} = \left[\frac{202}{9} \ln \frac{M_1}{M_U} - 87 \ln \frac{M_2}{M_U} + \frac{1159}{18} \ln \frac{M_3}{M_U}\right],$$

$$\Delta \ln \frac{M_U}{M_Z} = \left[\frac{101}{9} \ln \frac{M_1}{M_U} - 29 \ln \frac{M_2}{M_U} + \frac{305}{18} \ln \frac{M_3}{M_U}\right].$$

Using GUT threshold effects M_R can be lowered up to $\sim 10^{10}$ GeV while keeping M_U within the Planck scale.

Typical solutions

M_R (GeV)	M_U (GeV)	$\frac{M_1}{M_U}$	$\frac{M_2}{M_U}$	$\frac{M_3}{M_U}$	α_G^{-1}
5×10^9	1.58×10^{16}	2.204	1.200	0.659	15.0
10^{10}	1.58×10^{16}	2.065	1.160	0.659	15.0
10^{11}	1.58×10^{16}	1.661	1.050	0.656	15.0

GUT-scale threshold effect solutions (triplet model).

Obstruction in lowering of M_R below $\sim 10^9$ GeV:

α_{B-L} hits the Landau Pole

Non-renormalizable interactions

Gauge invariant but non-renormalizable terms could arise from quantum gravity

$$\mathcal{L}_{NRO} = -\frac{\eta_1}{2M_G} Tr\left(F_{\mu\nu}\Phi_{210}F^{\mu\nu}\right) - \frac{\eta_2}{2M_G} Tr\left(F_{\mu\nu}\Phi_{54}F^{\mu\nu}\right).$$

$\langle 0|\Phi|0\rangle \neq 0 \sim M_U$ produces kinetic-like gauge terms but with different strengths for the subgroups. Defi ning:

$$\epsilon_1 = \frac{3\eta_1}{4}\frac{M_U}{M_G}\left[\frac{1}{4\pi\alpha_G}\right]^{\frac{1}{2}}, \qquad \epsilon_2 = \frac{3\eta_2}{4}\frac{M_U}{M_G}\left[\frac{1}{15\pi\alpha_G}\right]^{\frac{1}{2}}$$

corrections to different gauge coupligs are:

$$\epsilon_{2L} = \epsilon_{2R} = -\frac{3}{2}\epsilon_2,\ \epsilon_{3C} = \epsilon_2 - \epsilon_1,\ \ \epsilon_{BL} = 2\epsilon_1 + \epsilon_2,$$

where in the RG eqs.,

$$\Delta_i^{(gr)} = -\frac{\epsilon_i}{\alpha_G}, i = BL, 2L, 2R, 3C.$$

Non-renormalizable interactions

Resulting corrections are:

$$\left(\Delta \ln \frac{M_R}{M_Z}\right)_{gr} = -\frac{\pi}{7\alpha_G}\left[\epsilon_1 + 10\epsilon_2\right], \quad \text{and} \quad \left(\Delta \ln \frac{M_U}{M_Z}\right)_{gr} = \frac{\pi}{7\alpha_G}\left[5\epsilon_2 - 3\epsilon_1\right].$$

Depending on η_i low M_R solutions can be found:

M_R (GeV)	M_U (GeV)	η_1	η_2	α_G^{-1}
10^9	3.16×10^{18}	0.305	0.96	25.00
10^7	3.16×10^{18}	0.494	1.16	25.64
10^6	8×10^{17}	2.728	4.77	25.32
10^5	3.16×10^{18}	0.671	1.34	25.32

Extra multiplets

Extra light-multiplets at the scale M_R can affect gauge coupling evolution.

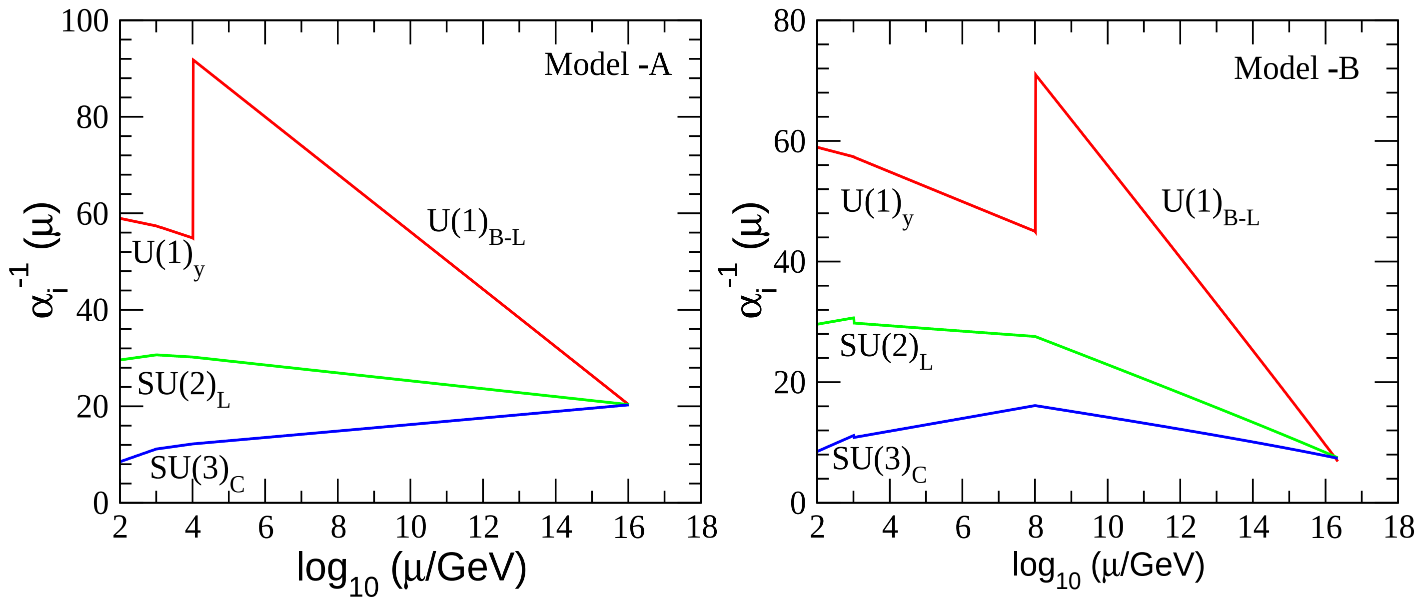
Two Models

Model	submultiplets	SO(10)
A	$\sigma(3,1,1,4/3) \oplus \overline{\sigma}(\overline{3},1,1,-4/3)$	**45, 210**
	$\eta(1,1,1,2) \oplus \overline{\eta}(1,1,1,-2)$	**120**
B	$\xi(6,1,1,4/3) \oplus \overline{\xi}(\overline{6},1,1,-4/3)$	**54,**
	$\eta(1,1,1,2) \oplus \overline{\eta}(1,1,1,-2)$	**120**
	A pair of $C(1,2,2,0)$	**10, 120, 126**
	$D_L(1,3,1,0) \oplus D_R(1,1,3,0)$	**45, 210**

Extra multiplets (contd.)

Model	M_R (GeV)	M_U (GeV)	α_G^{-1}
A	10^9	1.15×10^{16}	22.22
	10^5	1.10×10^{16}	20.83
	10^4	10^{16}	20.40
B	10^9	1.82×10^{16}	7.58
	10^8	2.00×10^{16}	10.13

Unification with extra multiplets



Conclusions

- M_R is the scale of Parity restoration (caution: D-parity)
- Low M_R is phenomenologically interesting
- Leptogenesis below the 'reheating' scale
- Possible avenues explored:
  1. GUT-scale threshold corrections
  2. Non-renormalisable interactions due to gravity
  3. Additional light multiplets
- Caveat: (15,1,3), (15,3,1) $\subset$ 210 can be light, upsetting results. Addition of SO(10) 54 may address this pitfall.

THANK YOU!